Classicality from zero discord for continuous variable bipartite systems

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Abstract. Quantum information resources quantified by non-zero discord are ubiquitous for the continuous-variables bipartite systems. Complementary to this, we formally construct a model characterized by zero two-way discord for arbitrarily long time interval. The model is classical in the sense it does not support quantum information processing. We point out some interesting physical features of the model.

1. Introduction

"Quantum discord" is a common term for different measures of non-classical correlations in composite (e.g. bipartite) quantum systems [1]. Historically the first and probably the best known is the so-called "one-way" discord (to be defined in Section 2) [2, 3]. The closely related "two-way" discord is even a more stringent criterion for classical correlations [1, 4].

Originally, quantum discord has been introduced as a criterion for the decoherence-induced classicality [2]: zero discord states are considered the main candidates for "classicality". Recent elaboration [4] mutually links the different definitions of decoherence and the related measures of discord as the conditions for classicality.

A recent analysis of the one-way-discord dynamics provides a remarkable observation [5]. The authors find [5]: "that for almost all states of positive discord, the interaction with any (non-necessarily local) Markovian bath can never lead either to a sudden, permanent vanishing of discord, nor to one lasting a finite time-interval". In effect, not only sudden death of discord cannot be expected, but Markovian dynamics only leads us *asymptotically close* to a zero-discord state. From this result one may possibly expect the Markovian bipartite systems are practically deprived of zero discord states and from the related classicality.

However, the analysis in [5] does not rule out that there can be zero discord *for all times*. If a state starts with zero discord, it could be zero discord for all times. In the more general context: complementary to Ferraro et al [5], Markovian dynamics may probably provide *non-asymptotic* zero-discord for a bipartite system in a *long* time interval (that we term '*Markovian classicality*').

In this paper we *construct* a model implementing the two-way-discord 'Markovian classicality' (Definition 1) for the continuous-variable systems. Physical relevance of the model stems from the variations of the composite system's degrees of freedom (structure) [6] of a bipartite system. So our approach is complementary to the standard one [1, 2, 3, 4, 5] (and the references therein) that considers a concrete structure (decomposition into subsystems) of a bipartite quantum system. We find that the model-structure of our considerations exhibits some basic features of the classical-systems' structures.

In Section 2 we precisely define our task and we derive the main result of this paper. Section 3 is Discussion and we conclude in Section 4.

2. The model

One-way quantum discord for the $S+S'$ system, $D^{\leftarrow}(S|S') = I(S:S') - J^{\leftarrow}(S|S') \geq 0$, and von Neumann entropy of a state ρ, $\mathcal{S} = -tr\rho \ln \rho$. Both the total mutual information, $I(S:S') = \mathcal{S}(S) + \mathcal{S}(S') - \mathcal{S}(S,S')$, and the classical correlations, $J^{\leftarrow}(S|S') = \mathcal{S}(S) - \inf_{\{\Pi_{S'i}\}} \sum_i |c_i|^2 \mathcal{S}(\rho_S|_{\Pi_{S'i}})$–where $\rho_S|_{\Pi_{S'i}} = I_S \otimes \Pi_{S'i} \rho I_S \otimes \Pi_{S'i}$ is the state remaining after a selective quantum measurement defined by the projectors $\Pi_{S'i}$–are non-negative. The CC states are the only states fulfilling the condition

$D^{\leftarrow}(S|S') = 0 = D^{\rightarrow}(S|S')$. As two-way discord tends to be larger than one-way discord [4], we consider the zero *two*-way-discord (the CC) states as the *classical* states.

Definition 1. An open quantum system, C, consisting of two subsystems, S and S', is said to bear *Markovian classicality* if it can be described by a classical-classical (CC) state in *long* time-interval. A CC state is of the form $\sum_{m,n} \omega_{mn} P_{Sm} \otimes \Pi_{S'n}$, where the real numbers $\omega_{mn} \geq 0$ and $\sum_{m,n} \omega_{mn} tr_S P_{Sm} tr_{S'} \Pi_{S'n} = 1$ for the projectors P_{Sm} and $\Pi_{S'n}$ on the respective Hilbert spaces.

For separable $\omega_{mn} = p_m q_n, \forall m, n$, such that $\sum_m p_m tr_S P_{Sm} = 1 = \sum_n q_n tr_{S'} \Pi_{S'n}$, one obtains the tensor-product states, $\rho_S \otimes \rho_{S'}$, as a special kind of CC states. Physically, the composite system C may be e.g. a pair "object of measurement + apparatus" or "the internal + the center-of-mass" degrees of freedom of the Brownian particle.

Our task is to answer the following question: is there a model of a bipartite system that can provide Markovian classicality? Thereby, Definition 1 directly sets the following constraint on constructing a Markovian classicality model:

Classicality Constraint: *Two-way quantum discord is exactly zero in every instant in time before eventual thermalization of the open system.*

Getting into details, we detect the following obstacles to construct a model fulfilling the Classicality Constraint. First, initial non-zero discord in $S + S'$ system; Second, interaction between S and S'; Third, the common environment, E, for S and S'; Fourth, non-completely positive dynamics for the S' system; Fifth, the initial non-tensor-product state for C and E; Sixth, arbitrary initial zero-discord state for C.

The origin of these obstacles is respectively as follows: First, an initial non-zero discord state cannot fulfil the classicality condition. e.g. The dynamic transition

$$\sum_i \lambda_i \rho_{Si} \otimes \rho_{S'i} \rightarrow \sum_{m,n} \omega_{mn} |m\rangle_S \langle m| \otimes |n\rangle_{S'} \langle n| \tag{1}$$

is not allowed as long as the rhs of Eq. (1) refers to a continuous time interval [5]. There are at least three ways for dynamically obtaining a non-zero-discord state: Interaction between S and S', the common environment for S and S', and non-completely positive dynamics for the open system S' [1,7]. Markovian dynamics requires the tensor product initial state $\rho_C \otimes \rho_E$ [8, 9]. Finally, in general, the external (e.g. experimentally uncontrollable) local influence can raise the initially zero discord [10]. The local operations exerted on S and/or on S', the rhs of Eq. (1), can give rise to non-zero-discord final state. The *only state immune* to this (yet for the completely positive dynamics) is actually the tensor-product state, $\rho_S \otimes \rho_{S'}$.

Bearing all this in mind, the *only* option we offer is the following model:

$$S + (S' + E) \tag{2}$$

where the subsystem S does not interact with any other subsystem (S' and E) while assuming Markovian and completely positive dynamics for the open system, S', and the tensor-product initial state $\rho_S \otimes \rho_{S'} \otimes \rho_E$ for the total system. In principle, both S and S' can be composite systems themselves.

Then the unitary operator for the total system separates as:

$$U(t) = U_S(t) \otimes U_{S'+E}(t) = \exp\{-\imath t H_S/\hbar\} \otimes \exp[-\imath t (H_{S'} + H_E + H_{S'E})/\hbar] \quad (3)$$

and provides unitary (the Schrödinger) dynamics for both the S system as well as for the $S' + E$ system. Markovian and completely positive dynamics of S' does not introduce any additional correlation for S and S'. Then for the model Eq. (2), one can write for the open system's state:

$$\rho_S(t) \otimes \rho_{S'}(t) \quad (4)$$

in every instant in time, where $\rho_S(t) = U_S(t)\rho_S(0)U_S^\dagger(t)$ and $\rho_{S'}(t)$ is a solution to a Markovian-type master equation. The proof of Eq. (4) obviously follows from Eq. (3).

From Eq. (4) it easily follows: $\mathcal{S}(S, S') = \mathcal{S}(S) + \mathcal{S}(S')$ and therefore the equalities $D^{\leftarrow}(S|S') = 0 = D^{\rightarrow}(S|S')$ in every instant in time. So, we can say *we have designed a model that fulfills the very tight conditions for non-asymptotic zero-discord classicality of a Markovian bipartite system*: (i) the model Eq.(2)-(4) is distinguished, and (ii) the open system's dynamics is a completely positive map.

3. Discussion

The model Eqs. (2)-(4) is *designed* so as to fulfill the Classicality Constraint. It therefore represents a *sufficient* condition for Markovian classicality. For the tensor-product initial state, $\rho_S \otimes \rho_{S'}$, the subsystems S and S' remain mutually exactly uncorrelated in every instant in time, Eq. (4). In terms of [5]: the composite system's state remains in the $\Omega_\circ$ set of zero-discord states, all the time. As $[\rho_S \otimes I_{S'}, \rho_{S+S'}] = 0 = [I_S \otimes \rho_{S'}, \rho_{S+S'}], \forall t$, the state $\rho_{S+S'}$ Eq. (4) is a "doubly" lazy state [11]. Thus, we point out a 'niche' for the bipartite system's Markovian classicality. In the remainder of this section, we answer the following question: is there any realistic physical system supporting the model Eq.(2)?

To support intuition, we call for the standard picture of the macroscopic (classical) bodies. The macroscopic systems we usually term "classical" are of course the many-particle systems (rarely in thermal equilibrium yet). The constituent particles are in mutual interaction thus not supporting the model Eq. (2). However, classicality of the macroscopic bodies actually does *not* refer to the constituent particles' degrees of freedom. It is a general appearance of the macroscopic bodies (that include the Solar system's planets and the Sun, the living organisms etc.): classical behavior refers to the *specific degrees of freedom*, i.e. to the formally *alternate subsystems*. Paradigmatic are the *collective* degrees of freedom: the center-of-mass (CM) and the internal (the "relative", R) degrees of freedom. In idealized considerations, the two subsystems, CM and R, are mutually uncoupled and typically only one of them is subject to the environmental influence (e.g. to decoherence) [8, 12].

By following this classical wisdom, we promote the following *approach to Markovian classicality*: perform a proper variables transformations on the constituent particles' degrees of freedom in order to obtain the structure Eq. (2); see Appendix A for some

technical details referring basically to the continuous variable (CV) systems. If this is not possible, then the concrete model does not bear Markovian classicality.

So we distinguish the main lesson of this paper
Markovian classicality is not *a matter of a composite system itself* but *is a matter of a composite system's* structure.

We emphasize some features of the model Eq. (2).

First, formally the same model appears in some recent considerations of non-Markovian dynamics of the open bipartite systems [13].

Second, the model Eq. (2) is in intimate relation to quantum information localization measured by "locally inaccessible information (LII)" flow [14], as well as with quantum discord saturation [15] and quantum decorrelation [16]. Actually:
Lemma 1. The following are mutually equivalent statements: (i) the system $S+S'+E$ is in the state Eq. (2), (ii) quantum discord $D^{\leftarrow}(S+S'|E) = \mathcal{S}(E)$ is saturated (maximal), (iii) there is total decorrelation of the S system from the system S' and (iv) there is quantum information localization in the $S'+E$ system.

The proof of this lema is given in Appendix B that distinguishes the physical relevance of the model Eq. (2). Saturation of quantum discord (in $S'+E$) is equivalent to locking information locally (in $S'+E$), i.e. to decorrelation of the rest (S) of the composite system. So, Markovian classicality of $S+S'$ coincides with quantumness of $S'+E$. Of course, external influence on $S'+E$ leads to the loss of maximum discord. Bearing in mind the result of Ferraro et al [5], cf. Introduction, Lemma 1 suggests the locking of information [14], discord saturation [15] and quantum decorrelation [16] are dynamically feasible *only asymptotically*.

Finally, in support to the physical relevance of the model Eq.(2), we emphasize: Regarding the CV *Gaussian* states, the results in [17] strongly support the model: the only Gaussian states bearing zero discord are–of the form Eq. (2). Of course, for non-Gaussian states, the things may look different.

4. Conclusion

We construct a model of a bipartite continuous-variable (CV) open system that does not support quantum information processing. We do not claim there are not alternate models implementing zero-discord of a Markovian system for arbitrarily long time interval. The main lesson stemming from our considerations reads: The zero-discord classicality of a bipartite CV system is not a characteristic of the system itself. Rather, it's a characteristic of a specific system's structure (decomposition into subsystems). The composite systems not describable by such structure are deprived of the Markovian zero-discord classicality.

Acknowledgments

The work on this paper is financially supported by Ministry of Science Serbia under contract no 171028.

Appendix A.

A composite system C consists of two subsystems, 1 and 2, with the *continuous* degrees of freedom and the related conjugate momentums, (x_1, p_1) and (x_2, p_2), respectively. The C's Hilbert space factorizes, $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$, and the Hamiltonian $H = H_1 + H_2 + H_{12}$; H_{12} is the interaction term. The open system C is assumed to be subjected to a Markovian environment E. The total system $C + E$ is subject to the Schrödinger law.

The linear canonical transformations (LCT) introduce the new (also continuous) degrees of freedom and the related conjugate momentums:

$$\begin{aligned} X_A &= \sum_i \alpha_i x_i, P_A = \sum_j \gamma_j p_j, \quad [X_A, P_A] = \imath\hbar \\ \xi_B &= \sum_m \beta_m x_m, \pi_B = \sum_n \delta_n p_n, \quad [\xi_B, \pi_B] = \imath\hbar \end{aligned} \tag{A.1}$$

that give rise to the constraints:

$$\sum_i \alpha_i \gamma_i = 1 = \sum_i \beta_i \delta_i, \quad \sum_i \alpha_i \delta_i = 0 = \sum_i \beta_i \gamma_i. \tag{A.2}$$

The LCTs do not change any of the environmental degrees of freedom yet redefine the composite system C. The formal subsystems A and B pertain to the tensor-factorization of the state space, $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, while the Hamiltonian obtains a new form $H = H_A + H_B + H_{AB}$.

We do not consider the 'separable' unitary transformations of the form $U_1 \otimes U_2$– these are known to preserve discord [1, 10] (and the references therein). We also do not account for the unitary transformations referring to the change of the reference frame or the relativistic transformations [18]. Finally, we are not interested in the formally trivial kinds of LCTs such as the particles permutations or (re)grouping, fine- or coarse-graining of the structure of the composite system. The LCTs of interest *mix* the subsystems' degrees of freedom and make the different structures mutually *irreducible*. Of course, every state ρ_C is unique in every instant in time. However, its forms pertaining to the different structures, $1 + 2$ and $A + B$, are not [6].

While both structures simultaneously evolve in time, Markovian dynamics of the $1+2$ structure need not be applicable to the alternate $A+B$ structure. In general, there is not a guarantee that the environment E is Markovian also for the $A + B$ structure. Likewise, the RWA (the secular) approximation valid for the $1 + 2$ structure need not be applicable to the $A + B$ structure. The LCTs providing separation of variables for the $A + B$ structure ($H_{AB} = 0$) can in principle provide decoupling of one subsystem, e.g. the B subsystem, from the environment.

As an illustration, we emphasize the paradigmatic "collective" observables of a many-particle system. For a many-particle system defined by the degrees of freedom, $\{x_i, i = 1, 2...N\}$, one can introduce the system's center of mass (CM) and the internal (the "relative", R) formal subsystems. In idealized considerations, the new subsystems, CM and R, are exactly mutually separated–compare to Eq.(2) of the body text. The LCTs providing the variables separation for CM and R can, in principle, give rise to decoupling of one of the new subsystems from the environment. This is the case e.g. in the standard model of the quantum Brownian motion [8] (and the references therein), where only CM degrees of freedom are subject to environmental influence.

The task implicit to Section 2 in the main text reads: For a composite system C (that still may be multiparticle), to provide a bipartite structure as defined by the model Eq. (2). If the initial state is tensor-product for *that* structure, then that structure is described by Markovian classicality, Definition 1.

Appendix B.

We prove Lemma 1 in a way supporting some intuition about the zero-discord classicality. The more formal and more simple proofs will be provided elsewhere.

If one assumes the pure initial states for both S' and E, then Eqs. (2)-(4) directly give for the total system's instantaneous state:

$$\rho_S \otimes |\Psi\rangle_{S'+E}\langle\Psi| \tag{B.1}$$

and *vice versa*. In Eq. (B.1), the S' and E systems are in entangled pure state; for $\rho_S^2 = \rho_S$, the ρ_S state is also pure. The state Eq. (B.1) is exact. The entanglement is due to the interaction $H_{S'E}$, see Eq. (3), i.e. due to the fact that the environment effectively monitors and purifies the S' system.

Proof of Lemma 1 [body text]: Bearing in mind (i) is equivalent to (ii) (cf. Theorem 1 in [15]), the proof can be given by proving (i) is equivalent to (iii) and to (iv). That (i) implies (iv) is easy obtained. The "locally inaccessible information" flow [14], $\mathcal{L}^{\leftrightarrow} = D^{\leftrightarrow}(S'|S) + D^{\leftrightarrow}(E|S') + D^{\leftrightarrow}(S|E) = D^{\leftrightarrow}(E|S')$; there is only information flow in $S' + E$ system. Now we prove the inverse to this implication. Due to non-negativity of discord, the above equality for $\mathcal{L}^{\leftrightarrow}$ directly implies $D^{\leftrightarrow}(S|S') = 0 = D^{\leftrightarrow}(S|E)$. As we know $D^{\leftrightarrow}(S'|E) \neq 0$, the condition $D^{\leftrightarrow}(S|S') = 0 = D^{\leftrightarrow}(S|E)$ can be satisfied only by the state Eq. (B.1); e.g., the alternative tripartite state, $\sum_i c_i |i\rangle_S |i\rangle_{S'} |i\rangle_E$, that satisfies $D^{\leftrightarrow}(S|S') = 0 = D^{\leftrightarrow}(S|E)$, does not satisfy $D^{\leftrightarrow}(S'|E) \neq 0$. Here (without loss of generality) we assume the total system $S + S' + E$ is subject to the Schrödinger law, cf. Eq. (3), and that the initial states of both S' and E are pure–thus the alternative mixed states are of no interest here. Finally, we prove equivalence of (i) and (iii). The decorrelation is defined [16] as a difference of the two total correlations in the initial and the final state, $I_{initial}(S : S') - I_{final}(S : S')$. For every initial state, decorrelation is maximal if $I_{final}(S : S') = 0$. So, we prove that $I_{final}(S : S') = 0$ is equivalent to Eq. (B.1). From Eq. (B.1) it directly follows: $I(S : S') = \mathcal{S}(S) + \mathcal{S}(S') - \mathcal{S}(S, S') = 0$. The

inverse is easily proved, as from $I(S:S')=0$ follows $\mathcal{S}(S,S')=\mathcal{S}(S)+\mathcal{S}(S')$, which, in turn, is fulfilled only for the product states, Eq. (2). By purifying the product state, Eq. (2), one obtains the state Eq. (B.1). This completes the proof.

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